Title: The Map Coloring Problem

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Subject: Mathematics

Level: 3-12

Type: Whole Class, Experiential (3 x 45 minute periods)

Objectives:

(a) General: The students explore the concept of map coloring. The students come to understand that the number of colors needed for a map is proportional to the number of shapes that all have the same neighbors.

(b) Specific:

After this lesson, students have:

1) Made a drawing of random lines in 2 dimensions
2) Colored in this drawing using the least amount of colors as possible so that no two shapes that share an edge have the same color. Note that shapes that share only a point may have the same color. See figure 1.
3) Drawn simple maps that can only be drawn with 2, 3, 4, and 5[1] colors. See figure two.

Materials:

1) Drawing paper
2) Rulers
3) Markers and crayons
4) A place to post completed drawings
5) Colored chalk for board examples

Procedures:

DAY 1

Direct students to make a drawing in the following way (see figure 1).

1) Use a ruler and pencil only.
2) Draw lines that go from one edge of the paper to the other only (no stopping in the middle).

[1] I have yet to find a map that needs five colors. I am beginning to suspect it it not possible. Perhaps one will be found or students can find a way to show that it is not possible.

3) Make at between 10 and 25 lines (as a general guideline).
4) Make lines that go in different directions.

Direct students to color in the drawing as follows.

1) Use the least amount of colors as possible.
2) If two shape share an edge, they cannot be the same color.
3) Two shapes that share point may share the same color.
4) Show examples and ask students if the could or could not use the same color for different shapes you draw on the board (see figure 3).
5) Check that students are following the directions.

DAY 2, 3

1) Discuss the drawing of the previous day. What strategies did students use to use the least amount of colors? Did anyone use a one color at a time strategy? Did anyone start at one point and move outward (this will work better - why?) or color randomly and fill in? Ask why some drawings only need 2 colors (draw a checkerboard).

2) Tell students you are now going to relax the rules for drawing. You may stop your lines in the middle of the paper (as long as it end on some line). Also, you may used curved lines.

_____3) Show an example of two and three color patterns. Use colored chalk. Ask students to draw 2, 3, 4, and 5 color drawings. Ask them to to draw different ones than the ones of the board.

3) Emphasize the difference between patterns that could be drawn with ***x*** amount of colors and those that **have to be drawn** with ***x*** amount of colors. For example, a checkerboard could be drawn with 4 colors but also could be drawn with less colors (namely, two). Many students will grapple with this concept at this point. Try to let them grapple with it and discover it for themselves. Students will come up and show you drawing. Show counter-examples of how their drawings could be drawn with fewer colors if need be. Encourage students to become experts at evaluating this for themselves and for each other.

4) Many students at point will figure that the more complex the drawing the more colors they need. At this point, emphasize showing **simple** ways for each number of colors.

5) Start asking the students the what the idea is. Help them discover that for a shape that needs ***n*** colors, each shape must have ***n-1*** neighbors such that each neighbor shares an edge with each of its ***n-1*** neighbors. See figure 4.

Evaluation:

_____1) Did each student produce a DAY 1 drawing? Did the use the least amount of colors for the drawing?

2) Were they discovering and discussing strategies for using the least amount of colors?

3) Did students produce drawings that required 2, 3, and 4 neighbors? It may be difficult for all students to do a 4 color drawings.

4) Did students get the concept of shared neighbors?

Variations:

_____1) Draw columns on a board for 2, 3, 4, 5 color drawing. Students can post their drawings as they discover them.

2) Older students can try to describe a general algorithm to use the least amount of colors.

3) Students can try and show (or prove) that a five color drawing is not possible? Would it be possible in three dimensions?

4) Extend the lesson to three dimensions.

Suggested Questions:

_____See text of lesson.

Background:

This problem is actually a rather famous problem in higher mathematics. It is called the graph coloring problem. Given an arbitrary undirected graph, what is the least of colors that can be used such that no neighbors share the same color? It turns out that this problem is NP-complete. An NP-complete

problem is one that can only be solved in non-polynomial time. That is, as the number of nodes in the graph increases, it takes a exponentially increased amount of time to solve the problem. In plain English, you or a computer has to exhaustively try all the possibilities to arrive at a guaranteed correct solution. However, some interested non-guaranteed heuristics are known. The students will discover some of these in day one. See Data Structures and Algorithms by Aho, Hopcroft, and Ullman, pages 3 to 10 for more information.